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MCDA Research Groups

Applied Mathematics Unit
Faculty of Law, Economics, and Finance
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http://sma.uni.lu/home

On December 7th, 2004, the Faculty of Law, Economics, and Finance of the newly founded University of Luxembourg (http://www.uni.lu), approved the creation of the Service de Mathématiques Appliquées (SMA, http://sma.uni.lu/home). This recent applied mathematics unit takes a closer look at the intervention of mathematics in the world of economics, finance, management and business information systems. Particular attention is given to recent mathematical theories of decision-making support, as well as to stochastic analysis and statistics used by economists and finance specialists in the resolution of statistical problems arising in the business world. The members of the SMA are at present : Raymond Bisdorff (professor, head of the SMA), Jean-Luc Marichal (assistant-professor), Jang Schiltz (assistant-professor), Patrick Meyer (assistant and doctoral student), and Claude Lamboray (doctoral student). The SMA hosted the 61st meeting of the EURO-MCDA Working Group, March 10-11, 2005, and recently ORBEL21, the 21st national conference of the Belgian OR society, January 17-19, 2007.

This presentation is focused on the SMA work that is relevant for the MCDA community:

Decision aiding methodology (méthodologie de l'aide à la décision)

MSC(2000): 05C20/69/85, 62C12, 62P20, 68R10, 90B50, 90C09/27, 91A30/35, 91B06/08/10/12/14716, 94C15

Our original contribution to the European school of Operational Research concerns the logical foundation of decision aiding methodology (Bisdorff 2000). We have shown the necessity to choose a non standard bipolar valued logical evaluation domain in order to avoid well known inconsistencies of classical multi-valued or fuzzy logics (Bisdorff 2002a). In directing our investigations early on to what the exceptional new algorithmic approaches of artificial intelligence brought to operational research (Bisdorff and Laurent 1995), we have been able to extend the application field of the graph kernel concept (independent and dominating set) to bipolar valued digraphs. This allows us to apply it to multicriteria ranking problems (Bisdorff, 1999), to multicriteria clustering problems (Bisdorff 2002b) and to choice decision problems (Bisdorff and Roubens, 2004). We have also investigated the notion of ordinal concordance of preferential assertions in the context of multicriteria preference modelling. This allowed us to introduce a specific robustness analysis of multicriteria decision aid recommendations (Bisdorff, 2004).

The acquisition in December 2004 of a new high performance application server gave us interesting perspectives of algorithmic development and new outstanding computing performances have been achieved. Indeed, we discovered recently a set of fundamental formal results, relating the valued choice approach with the correspon ding valued characteri stic vector approach (Bisdorff, Pirlot, Roubens 2006). These results give us by the way some clews for the development of new well performing valued kernel extraction algorithms. Actually we are able to extract valued kernels from a 75% filled random outranking graph of order 2500 in less than 30 seconds (Bisdorff, 2006, 2007).

A constant research effort concerns the *choice* problematique in multiple criteria decision aiding. Its genuine purpose is to help a decision maker determining a single best decision alternative. Methodologically we focus on pairwise comparisons of these alternatives which lead to the concept of *bipolar-valued* outranking digraph. This work is situated in the context of *progressive* decision aiding methods consisting normally in several stages providing the decision maker with more and more precise choice recommendations. The choice method developed in the Applied Mathematics Unit is called *Rubis* (Bisdorff et al. 2007). Its backbone is a bipolar-valued credibility scale, modelling the credibility of the validation of preferential statements. Its development leads us to define the concept of *hyperkernel* of a digraph as a choice recommendation. Lately we have extended this methodology to the k-choice problematique (Meyer and Bisdorff, 2007).

Claude Lamboray, for his part, is investigating the use of prudent ranking rules in ordinal aggregation problems. A prudent order is a ranking solution initially proposed by Arrow and Raynaud. It is based on minimizing the pairwise strongest opposition, which makes especially sense in a decision aid context. In order to gain a better understanding of the prudent approach, we have built an axiomatic framework which can be used to characterize prudent ranking rules (Lamboray 2006). We have also compared the prudent order solution to the solutions found by other common ranking rules encountered in the Social Choice literature. We are currently working on an extension of the prudence principle to exploit bipolar valued outranking relations in the ranking problematique.

In the framework of multiple attribute value theory (MAVT), Patrick Meyer furthermore contributes to the development of the Kappalab toolbox for the manipulation of non-additive integrals (http://www.polytech.univ-nantes.fr/kappalab, Grabisch et al. 2005, 2007). Kappalab, which stands for "laboratory for capacities", is a package for the GNU R statistical system. It is a toolbox for capacity (or non-additive measure, fuzzy measure) and integral manipulation on a finite setting which can be used in the framework of decision making or cooperative game theory. Kappalab contains routines for handling various types of set functions such as games or capacities. It can be used to compute several non-additive integrals: the Choquet integral, the Sugeno integral, and the symmetric and asymmetric Choquet integrals. An analysis of capacities in terms of decision behavior can be performed through the computation of various indices such as the Shapley value, the interaction index, the orness degree, etc. The well-known Möbius transform, as well as other equivalent representations of set functions can also be computed. Kappalab further contains seven capacity identification routines: three least squares based approaches, a method based on linear programming, a maximum entropy like method based on variance minimization, a minimum distance approach and an unsupervised approach grounded on parametric entropies.

Data Aggregation

MSC (2000): 26B35, 26E60, 28E10, 39A12, 39B12/22/72, 90B50, 90C29, 91A12, 91B06/08/12/14/16, 91C05, 91E45, 94A17.

The research topic stems form the area of data aggregation, domain more and more present in various disciplines of applied mathematics, with applications in decision making, cooperative game theory, theory of means and averages, and theory of functional equations, to name a few. Here, we are interested in real functions that aggregate numerical readings into one representative value. Various aggregation functions and processes have already been proposed in the literature and many others are still to be designed to fulfill newer and newer requirements. Studies on the aggregation problem have shown that the choice of the aggregation function is far from being arbitrary and should be based upon properties dictated by the framework in which the aggregation is performed. One of the main concerns when choosing an appropriate function is to take into account the scale types of the variables being aggregated. On this issue, it is now well known that the general form of the functional relationship between variables is greatly restricted if we know the scale types of the dependent and independent variables. For instance, if all the variables define a common ordinal scale, it is clear that any relevant aggregation function cannot be constructed from usual arithmetic operations, unless these operations involve only order. Thus, computing the arithmetic mean is forbidden whereas the median or any order statistic is permitted.

The research we have led thus far have been focused on the following themes:

Means and typical values. The most often encountered aggregation functions are similar to means or medians. Some extensions, like nonstrict or nonsymmetric means have been proposed and axiomatized. Some of them, like the ordered weighted averages, belong to more general families, called nonadditive or fuzzy integrals (Fodor and Marichal, 1997, Marichal, 2000a).

Nonadditive measures and integrals. The study of aggregation functions quickly led to the investigation of families of particular functions, namely nonadditive integrals such as the Choquet integral and the Sugeno integral. Those integrals represent a kind of average or median whose weights depend on the aggregated values. Their interest is the taking into account of the possible interaction between aggregated variables or attributes, what the classical means cannot model. We have contributed to the representation and the axiomatization of those particular functions (Marichal, 2000a, 2001).

Set functions, importance and interaction indices. Nonadditive integrals are constructed from nonadditive measures. In turn, these measures have been deeply investigated and have given rise to some indices, inspired from game theory, allowing us to measure the overall importance of any combination of attributes or the average degree of interaction among those attributes (Grabisch, Marichal, and Roubens, 2000). In this framework, we have introduced and axiomatized a dispersion index, called generalized entropy (Kojadinovic, Marichal, and Roubens, 2005), as well as tolerance indices measuring the tolerant or intolerant character of an aggregation function (Marichal, 2007).

Conjoint measurement. Conjoint measurement concerns the taking into account of the scales of measurement of aggregated variables. In this framework, we have particularly studied the case of ordinal scales, thus axiomatizing the sole functions allowed to aggregate purely ordinal information (Marichal and Mesiar, 2004).

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